IOWA MATHEMATICS
PERFORMANCE LEVEL DESCRIPTORS

General Performance Level Descriptors

The General Performance Level Descriptors (PLDs) provide descriptions of what students at each performance level *know* and what they *are able to do*. Taken together with Grade- and Content-specific PLDs and threshold scores, they convey the meaning of the ISASP results.	
Advanced	Students performing at the Advanced level demonstrate thorough competency over the knowledge, skills, and abilities that meet the requirements for their grade level associated with academic readiness for college and career in the subject area.
Proficient	Students performing at the Proficient level demonstrate adequate competency over the knowledge, skills, and abilities that meet the requirements for their grade level associated with academic readiness for college and career in the subject area.
Not-Yet-Proficient	Students performing at the not-yet-proficient level have not yet demonstrated the knowledge and skills to be classified as Proficient.

Using the Performance Level Descriptors

The Iowa Mathematics Performance Level Descriptors (PLDs) outline a range of skills a Proficient or Advanced student could demonstrate. The descriptors are representative of Proficient and Advanced skills at a given grade but are not meant as an exhaustive list of all possible content that could be assessed on an ISASP Mathematics form.
Descriptors in the Advanced category include skills a student could demonstrate that go beyond the scope of Proficiency. The categories are cumulative in that an Advanced student should consistently demonstrate the skills within the Proficient category as well as one or more of the skills within the Advanced category.
All limitations noted in the Iowa Core, such as the specific denominators used in Grades 3 and 4 Number & Operations – Fractions, apply to expectations in these Performance Level Descriptors.

Iowa Mathematics PLDs – Grade 3

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 390 to 442***	***ISASP Mathematics*** ***Scale Score Range: 443 to 510***
3.NBT **Number & Operations in Base Ten**	**A typical student at this level can:** • identify the place values of digits in the ones, tens, hundreds, and thousands places • round a 2- or 3-digit whole number to the nearest 10, a whole number with up to 4 digits to the nearest 100, or a single whole number to both the nearest 10 and the nearest 100 • apply strategies for rounding in a simple multistep problem • multiply 1-digit whole numbers by 10 or multiples of 10 • add and subtract 2 or more whole numbers with up to 3 digits within 1,000 with or without composing or decomposing tens and hundreds • represent addition and subtraction of whole numbers within 1,000 with models, such as base ten blocks	**A typical student at this level can:** • determine what numbers satisfy a set of conditions involving concepts of rounding • apply strategies for rounding in a complex multistep problem • multiply 1-digit whole numbers by multiples of 100 • evaluate strategies for solving a given addition or subtraction equation • identify errors in a solution strategy for a given addition or subtraction equation • interpret a context to add, subtract, and multiply by multiples of 10 to solve a multistep problem
3.NF **Number & Operations - Fractions**	**A typical student at this level can:** • understand a unit fraction as an equal part of a whole • represent unit fractions on a number line • write a fraction to represent a quantity in a simple context • interpret fractions in terms of equal parts of a whole and intervals on a number line • recognize fractional equivalence supported by visual models • compare 2 fractions with the same numerator or the same denominator with words or symbolically using <, >, or = • recognize that comparisons of fractions are valid only when the 2 fractions refer to the same whole	**A typical student at this level can:** • interpret and apply fractions, unit fractions, and fractional equivalence in terms of equal parts of a whole and intervals on a number line in a context involving more than 1 of the same whole • represent a whole number as a fraction and recognize fractions that are equivalent to whole numbers • create a model to demonstrate fractional equivalence • interpret a context requiring a comparison of 2 fractions of 2 different wholes • compare and order multiple fractions with the same numerator or the same denominator, or by generating equivalent fractions

Iowa Mathematics PLDs – Grade 3

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 390 to 442***	***ISASP Mathematics*** ***Scale Score Range: 443 to 510***
3.OA **Operations & Algebraic Thinking**	**A typical student at this level can:** • find an unknown in a multiplication equation • write or identify a product or quotient of whole numbers that describes equal groups of objects • solve 1-step and simple 2-step word problems • apply a property of operations to multiply or divide, or to find an unknown in a 1-step multiplication or division equation • understand division as an unknown-factor multiplication problem • calculate whole-number products and quotients in a context • multiply and divide within 100 • apply operations to solve simple 2-step problems • extend the terms of a simple arithmetic pattern • identify rules or simple characteristics of arithmetic patterns	**A typical student at this level can:** • solve 2-step and multistep word problems • apply multiple properties of operations to multiply and divide, or to find an unknown in a 2-step equation • interpret the meaning of whole-number products and quotients as it relates to a context • explain or defend rules for arithmetic patterns • identify a rule for an arithmetic pattern in a context, apply the rule to generate values, and interpret the values in terms of the context • interpret a complex context involving multiplication and division within 100 to solve problems
3.G **Geometry**	**A typical student at this level can:** • identify shapes with a given unit fraction shaded, when options are or are not partitioned into the same number of equal areas • identify shapes that are partitioned into equal and unequal areas • identify the unit fraction associated with a shape partitioned into equal areas • partition shapes into equal areas, and relate each part to a unit fraction or multiple parts to a fractional part • create examples and non-examples of shapes that fit into different categories • identify a shape, such as a quadrilateral, rectangle, square, or rhombus • identify shapes (quadrilateral, rectangle, square, rhombus) in a figure composed of multiple shapes	**A typical student at this level can:** • compare sizes of shaded fractions of 2 same-size shapes divided differently, such as different number or orientations of equal parts • relate a unit square to a fraction of the area of a rectangle, given the lengths of the sides • understand a shape as being in a subcategory of another shape due to shared attributes • describe a relationship between a category and subcategory of shapes

Iowa Mathematics PLDs – Grade 3

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 390 to 442***	***ISASP Mathematics*** ***Scale Score Range: 443 to 510***
3.MD **Measurement & Data**	**A typical student at this level can:** • tell and write expressions of time • measure time intervals in minutes • estimate length • measure length, including a 1-step measurement with a ruler with 1 end or neither end of the object at zero • measure liquid volume and mass • solve 1- and 2-step problems with time, length, liquid volume, and mass with whole numbers • solve 1-step problems using unit-scaled pictographs, bar graphs, and line plots • create and interpret pictographs, bar graphs, and line plots, and use them to solve 1- and 2-step problems • find perimeters, given the side lengths of polygons • solve problems related to perimeter, including finding a missing side length, given the perimeter • identify a unit square • find the area of a rectangle by counting unit squares or multiplying whole-number side lengths • find the area of a tiled shape	**A typical student at this level can:** • solve problems with time, length, liquid volume, and mass with simple fractions • solve multistep problems with time, length, liquid volume, and mass with whole numbers • explain or defend a measurement, calculation, or interpretation • solve multistep problems involving interpreting scaled pictographs, bar graphs, and line plots • recognize patterns involving the relationship between area and perimeter • find areas by decomposing figures • relate multiple ways of finding the area of a rectangle or rectilinear shape that is tiled in unit squares • estimate the area of a non-rectilinear shape that is tiled in unit squares

Iowa Mathematics PLDs – Grade 4

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 409 to 475***	***ISASP Mathematics*** ***Scale Score Range: 476 to 540***
4.NBT **Number & Operations in Base Ten**	**A typical student at this level can:** • estimate and round numbers to specified place values, including to the nearest 10, 100, or 1,000 • recognize a digit in one place represents 10 times as much as it represents in the place to the right • read, write, and compare numbers in standard form up to 1,000,000 or multi-digit numbers in expanded notation • add and subtract up to 1,000,000 • multiply a 2-, 3-, or 4-digit whole number by a 1-digit whole number or two 2-digit whole numbers • divide a 2-, 3-, or 4-digit whole number by a 1-digit whole number, including identifying remainders	**A typical student at this level can:** • reason quantitatively about the directional characteristics of place value, recognizing a digit in one place having a value of a multiple or quotient of 10 (100, 1,000, and 10,000) times as much as in a place to the right or left • identify efficient strategies for adding or subtracting multi-digit whole numbers • identify and correct errors in a given strategy for adding or subtracting multi-digit whole numbers • apply concepts of remainders in division problems • illustrate and explain calculations when multiplying and dividing
4.NF **Number & Operations - Fractions**	**A typical student at this level can:** • compare 2 fractions with like numerators, like denominators, or different denominators • create and represent equivalent fractions • compare 2 fractions symbolically by using <, >, and = • identify tenths and hundredths, both as fractions and as decimals, by using visual models • express and represent equivalence between fractions with denominators of 10 and 100 • represent and decompose fractions as a sum of unit fractions • solve 1- and 2-step problems with addition and subtraction of fractions with like denominators • solve 1- and 2-step problems with multiplication of fractions by whole numbers • compare 2 or 3 decimal numbers that are all to the same place or 2 decimal numbers that are to different places, up to the hundredths	**A typical student at this level can:** • apply, represent, and explain fraction equivalence • order more than 2 fractions with different denominators • add and subtract fractions and mixed numbers with like denominators • solve multistep problems with addition and subtraction of fractions with like denominators • represent and explain multiplication of fractions by whole numbers • solve multistep problems with multiplication of fractions by whole numbers • compare 3 or more decimal numbers that are to different places, up to the hundredths

Iowa Mathematics PLDs – Grade 4

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 409 to 475***	***ISASP Mathematics*** ***Scale Score Range: 476 to 540***
4.OA **Operations & Algebraic Thinking**	**A typical student at this level can:** • represent verbal statements involving multiplication as equations, such as writing "12 is 3 times as much as 4" as 12 = 3 × 4 • distinguish between additive and multiplicative comparisons • create or use an equation with multiplication or division and a symbol for the unknown to solve problems • solve 1-, 2-, or multistep word problems using the 4 operations with whole numbers, including interpreting remainders or estimating • find factor pairs to 100, and identify multiples up to 100 of a given 1-digit number • identify the next term in a number or shape pattern • generate number and shape patterns that follow a given rule • determine and apply a simple rule for a number or shape pattern • determine whether a whole number up to 100 is prime or composite	**A typical student at this level can:** • solve multistep word problems using the 4 operations, including interpreting remainders or estimating • find prime factors of a complex number • generate a rule for a given number or shape pattern, including a rule expressed algebraically • explain the application of a rule for a number or shape pattern • identify features of a pattern not explicit in the rule itself • explain the difference between prime and composite numbers
4.G **Geometry**	**A typical student at this level can:** • identify right triangles • draw or identify points, lines, angles (acute, right, obtuse), line segments, rays, and parallel and perpendicular lines in simple 2-dimensional figures • identify or draw lines of symmetry in simple 2-dimensional figures • complete a drawing of a 2-dimensional figure given a line of symmetry • classify quadrilaterals based on the presence or absence of parallel or perpendicular lines	**A typical student at this level can:** • draw or identify lines, line segments, rays, angles (acute, right, obtuse), and parallel and perpendicular lines in complex 2- dimensional figures or simple 3-dimensional figures • provide examples of 2-dimensional figures when given multiple, specific characteristics, including parallel and perpendicular lines and angles of a specified size • explain why a triangle is acute, right, or obtuse • explain why a quadrilateral is a parallelogram, rhombus, or rectangle • explain why a given figure has or does not have a line of symmetry • draw lines of symmetry in complex 2-dimensional figures • draw a 2-dimensional figure that has a given number of lines of symmetry

Iowa Mathematics PLDs – Grade 4

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 409 to 475***	***ISASP Mathematics*** ***Scale Score Range: 476 to 540***
4.MD **Measurement & Data**	**A typical student at this level can:** • solve 1- and 2-step problems, including 1- and 2- step conversions, involving time, length, mass, capacity, and money with the 4 operations on whole numbers and simple fractions and decimals • find the areas and perimeters of rectangles in real-world and mathematical problems • find a dimension of a rectangle given the area or perimeter and the other dimension • identify data from line plots in fractional units • draw line plots to represent data in fractions of a unit • solve 1- and 2-step problems involving the interpretation of data on a line plot, including operations on whole numbers and proper fractions • measure angles with a protractor • identify or draw angles with given measures • solve 1- and 2-step addition and subtraction problems involving angles	**A typical student at this level can:** • solve multistep problems, including measurement conversions, using the 4 operations • solve multistep problems involving the interpretation of data on a line plot, including operations on mixed numbers • solve multistep addition and subtraction problems involving angles in a complex figure • interpret an angle in a circle as a fraction of 360 degrees

Iowa Mathematics PLDs – Grade 5

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 429 to 502***	***ISASP Mathematics*** ***Scale Score Range: 503 to 590***
5.NBT **Number & Operations in Base Ten**	**A typical student at this level can:** • identify the place value name to the thousandths • recognize a digit in one place represents 10 times or 1/10 as much as it represents in the place to the right or left • reason quantitatively about the directional characteristics of place value • compare decimal numbers that are to the same place or different places, up to the thousandths • round decimal numbers to thousandths • multiply and divide by powers of 10 written as whole numbers (10, 100, 1,000, etc.) • multiply or divide by a single power of 10 written using a whole-number exponent • evaluate powers of 10 using whole-number exponents • represent numbers that are powers of 10 using whole-number exponents • multiply and divide multi-digit whole numbers • add, subtract, multiply, and divide decimals to the hundredths • apply operations on whole numbers and decimal numbers to solve problems	**A typical student at this level can:** • explain patterns relating to the directional characteristics of place value • compare 3 or more decimals to any place, including in expanded form • multiply and divide by 2 or more powers of 10 written using whole-number exponents • round decimals to any place • illustrate or explain division of multi-digit whole numbers using arrays or models • apply, illustrate, or explain strategies used to perform operations on decimals

Iowa Mathematics PLDs – Grade 5

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 429 to 502***	***ISASP Mathematics*** ***Scale Score Range: 503 to 590***
5.NF **Number & Operations - Fractions**	**A typical student at this level can:** • solve problems with addition and subtraction of whole numbers, proper and improper fractions, and mixed numbers with like and unlike denominators with or without regrouping, and with or without models • multiply a proper fraction by a whole number • solve problems with multiplication of a proper fraction by a proper or improper fraction • use models to represent and solve division problems involving proper fractions • interpret a fraction as a division problem • solve 1- and 2-step problems that involve the division of whole numbers and that lead to answers in the form of fractions or mixed numbers • determine the value of an unknown in a simple equation relating 3 proper fractions	**A typical student at this level can:** • solve problems with multiple decision points involving addition, subtraction, and multiplication of fractions with unlike denominators • fluently multiply or divide a proper or improper fraction, mixed number, or whole number by a mixed number • represent and solve division problems involving fractions without using models • solve multistep problems that involve the division of whole numbers and that lead to answers in the form of fractions or mixed numbers • determine the value of an unknown in an equation relating proper or improper fractions, whole numbers, or mixed numbers • solve problems by applying general concepts of fractions without specific values

Iowa Mathematics PLDs – Grade 5

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 429 to 502***	***ISASP Mathematics*** ***Scale Score Range: 503 to 590***
5.OA **Operations & Algebraic Thinking**	**A typical student at this level can:** • translate between words and symbols for a 1-, 2-, or multistep numerical expression of a calculation with numbers, including using grouping symbols • interpret a mathematical context to write a 1- or 2-step numerical expression of calculations with numbers, including using grouping symbols • evaluate a 1-, 2-, or multistep numerical expression, including using grouping symbols • identify and correct a mistake in a 1- or 2-step numerical expression or in the steps used to evaluate a multistep numerical expression, including using grouping symbols • identify or generate a rule for a given pattern • generate numerical patterns from rules for 2 patterns	**A typical student at this level can:** • translate between words and symbols for a complex numerical expression of calculations with numbers, including using multiple levels of grouping symbols • interpret a real-world context to write a numerical expression of calculations with numbers, including using grouping symbols • evaluate a complex numerical expression, including using multiple levels of grouping symbols • explain, defend, or correct a multistep numerical expression or the steps used to evaluate a complex numerical expression • explain the corresponding relationships between 2 patterns • translate numerical patterns into ordered pairs, and explain relationships in resulting data displayed on a coordinate plane
5.G **Geometry**	**A typical student at this level can:** • identify ordered pairs of points graphed in Quadrant I of a coordinate plane • identify or graph points in Quadrant I of a coordinate plane, including points on axes • describe the meaning of an ordered pair in a context • apply operations on values in a context represented by points graphed in Quadrant 1 • identify and classify 2-dimensional figures with given attributes, including identifying a square as a rectangle	**A typical student at this level can:** • interpret and describe patterns and relationships between ordered pairs of points in a context • interpret and apply a point or figure graphed in Quadrant I to solve mathematical or real-world problems • understand that a 2-dimensional shape in a subcategory has all the attributes of that category • apply or create a hierarchy to represent relationships among 2-dimensional figures

Iowa Mathematics PLDs – Grade 5

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 429 to 502***	***ISASP Mathematics*** ***Scale Score Range: 503 to 590***
5.MD **Measurement & Data**	**A typical student at this level can:** • calculate 1- and 2-step conversions of time, length, mass, and capacity with whole numbers and simple fractions and decimals • create and interpret line plots with fractions of a unit, and use information from line plots to solve problems • distinguish between perimeter, area, and volume • find the volume of a right rectangular prism, including by counting cubes and by using a formula	**A typical student at this level can:** • calculate multistep conversions of time, length, mass, and capacity, including multiplying a mixed number by a whole number or mixed number • interpret a context to solve problems involving multistep conversions of time, length, mass, and capacity • interpret multiple characteristics of line plots, and use information from line plots to solve complex problems • find the volume of a 3-dimensional solid composed of right rectangular prisms • given the volume and certain dimensions of a right rectangular prism, determine other dimensions

Iowa Mathematics PLDs – Grade 6

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 450 to 531***	***ISASP Mathematics*** ***Scale Score Range: 532 to 640***
6.NS **The Number System**	**A typical student at this level can:** • solve problems involving division of fractions • identify common factors and common multiples • find and apply least common multiples and greatest common factors • compute fluently (perform all 4 arithmetic operations) with multi-digit whole numbers and multi-digit decimals • order positive and negative integers on a number line • identify the opposites and absolute values of positive and negative integers • represent absolute values of rational numbers as distance from zero on a number line • identify integer points in all 4 quadrants • solve problems involving plotting integer points in all 4 quadrants • interpret the meaning of points in all 4 quadrants in terms of the context • represent and compare quantities in real-world contexts using rational numbers • interpret and create statements of order, magnitude, and comparison, including inequalities, relating rational numbers • find the distance between 2 points in any quadrants with the same 1st or the same 2nd coordinate	**A typical student at this level can:** • interpret a context using multiplication and division to divide decimals by decimals and fractions by fractions • solve problems involving rational-number points in all 4 quadrants • interpret and explain statements of order, magnitude, and comparison, including inequalities, relating rational numbers • use definitions, properties, expressions, and equations to defend solutions in problems • understand that ordered pairs differing only in signs are related by reflections across axes

Iowa Mathematics PLDs – Grade 6

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 450 to 531***	***ISASP Mathematics*** ***Scale Score Range: 532 to 640***
6.EE **Expressions & Equations**	**A typical student at this level can:** • evaluate expressions with whole-number exponents • read, write, and evaluate simple numerical and variable expressions • apply order of operations to evaluate multistep numerical and variable expressions • apply properties of operations, including the distributive property with whole numbers, to write equivalent expressions • solve 1-step 1-variable equations with whole numbers and rational numbers • use substitution to determine whether a given number makes a 1- or 2-step equation or inequality true • identify a simple 1-variable expression or 2-variable equation that models a context • create and graph inequalities on a number line • represent and model relationships between 2 variables in mathematical and real-world contexts with tables and graphs	**A typical student at this level can:** • interpret expressions, equations, and inequalities in real-world contexts • apply properties of operations to write equivalent expressions, including terms with whole-number exponents, using the distributive property with rational numbers, and factoring out the greatest common factor • interpret and analyze relationships between 2 variables in real-world contexts • fluently translate among representations, (graphs, tables, and equations) of relationships between 2 variables
6.RP **Ratios & Proportional Relationships**	**A typical student at this level can:** • represent ratios as part-to-part and part-over-part relationships • describe a ratio relationship between 2 quantities • identify a unit rate in a context without calculating • apply ratio concepts as numerical comparisons using division • determine equivalent rates and ratios • use ratio and rate reasoning to solve 1- and 2-step problems, including calculating and applying a unit rate • relate whole-number percentages to fractions out of 100 • find a percentage of a number • perform simple unit conversions	**A typical student at this level can:** • use ratio and rate reasoning to: • solve multistep problems • interpret the ratio relationship between quantities • perform multistep unit conversions • find the whole, given a part and a percentage

Iowa Mathematics PLDs – Grade 6

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 450 to 531***	***ISASP Mathematics*** ***Scale Score Range: 532 to 640***
6.G **Geometry**	**A typical student at this level can:** • find the areas of triangles, special quadrilaterals, and polygons composed of triangles and rectangles • solve 1- and 2- step word problems involving the areas of polygons composed of triangles and rectangles • use the formula to identify an expression of the volume of a right rectangular prism with rational numbers • use the formula to calculate the volume of a right rectangular prism with whole-number or up to 2 rational-number edge lengths • draw polygons in the coordinate plane, given coordinates for the vertices • determine the coordinates of the 4th vertex of a rectangle, given 3 vertices • find the length of a side of a polygon in any quadrants with the same 1st or the same 2nd coordinate • identify 3-dimensional objects represented as nets • represent 3-dimensional figures with nets • find the surface area, given a net, of a 3-dimensional object with sides in the shapes of triangles and special quadrilaterals • solve 1- and 2-step real-world problems involving surface areas • apply the volume of a right rectangular prism to solve problems with whole numbers or up to 2 rational numbers	**A typical student at this level can:** • solve multistep real-world problems involving the areas of polygons composed of triangles and rectangles • use the formula to calculate the volume of a right rectangular prism with rational edge lengths or edge lengths given as variables • solve multistep real-world problems involving surface areas • apply the volume of a right rectangular prism to solve multistep problems with rational numbers

Iowa Mathematics PLDs – Grade 6

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 450 to 531***	***ISASP Mathematics*** ***Scale Score Range: 532 to 640***
6.SP **Statistics & Probability**	**A typical student at this level can:** • identify a statistical question • interpret data presented in a table • determine the mean, median, mode, and range of any data set presented in a list or table • determine the median, range, first quartile, third quartile, and interquartile range of a data set presented in a box plot • report the total number of data points in a set presented in a line plot • describe the distribution of data in terms of shape, center, and spread • display data in line plots, histograms, and box plots • report basic facts about a data set presented in a histogram, such as the number of data points in a given range or in total • understand the difference between measures of center and measures of variation	**A typical student at this level can:** • analyze the effects to the mean, median, mode, and range when data are added to or taken away from a set • determine and explain the most appropriate measure of center and the most appropriate measure of variation based on the shape of the data and the context of the problem • interpret the mean, median, mode, and range in the context of data presented in a histogram or box plot • determine the mean absolute deviation of a data set • determine the mean, median, and mode of a data set presented in a frequency table

Iowa Mathematics PLDs – Grade 7

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 469 to 574***	***ISASP Mathematics*** ***Scale Score Range: 575 to 680***
7.NS **The Number System**	**A typical student at this level can:** • convert a fraction to a decimal using long division • use all 4 arithmetic operations with integers • use all 4 arithmetic operations with positive and negative rational numbers, including creating equivalent fractions • recognize additive inverses, rules for signs, absolute values, and properties of operations, and use them to solve mathematical and real-world problems with rational numbers	**A typical student at this level can:** • use all 4 arithmetic operations with rational numbers to solve multistep real- world problems, using fractions and decimals interchangeably, including translating among multiple representations of rational numbers • interpret the sum, difference, product, and quotient of rational numbers in a real-world context
7.EE **Expressions & Equations**	**A typical student at this level can:** • use properties of operations to generate equivalent expressions • apply the distributive property to generate an equivalent expression with integers • rewrite a numerical expression in a different form to show how quantities are related • use variables to represent quantities in expressions, equations, and inequalities, and use them to solve problems • solve 1- and 2-step equations and inequalities with integer and rational coefficients and solutions, assess the reasonableness of solutions, and graph the solutions • solve 1- and 2-step problems posed with integers • solve multistep problems posed with positive rational numbers in any form	**A typical student at this level can:** • use multiple properties of operations to generate equivalent expressions • apply the distributive property to generate an equivalent expression with rational numbers • rewrite an algebraic expression in a different form to show how quantities are related • solve multistep problems posed with rational numbers in any form • use variables to represent quantities in complex expressions, equations, and inequalities to solve problems • interpret solutions in context, including graphs

Iowa Mathematics PLDs – Grade 7

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 469 to 574***	***ISASP Mathematics*** ***Scale Score Range: 575 to 680***
7.RP **Ratios & Proportional Relationships**	**A typical student at this level can:** • identify a unit rate shown in a table or on a graph • calculate with whole numbers and fractions to determine a unit rate from a verbal description • use unit rates to solve problems • solve a given equation of a proportional relationship • identify and analyze proportional relationships by recognizing equivalent ratios • recognize a proportional relationship shown in a graph • identify specified points on the graph of a proportional relationship, and describe the relationship in terms of the context • solve mathematical and real-world problems with percentages	**A typical student at this level can:** • calculate and apply a unit rate in a complex context with multiple decision points • represent and calculate unit rates with ratios of fractions • set up and solve an equation of a proportional relationship to solve problems • analyze and interpret proportional relationships, and use them to solve complex, multistep problems by comparing rates and ratios, determining and applying rates, and determining rates from graphs and equations • solve complex, multistep mathematical and real-world problems with percentages

Iowa Mathematics PLDs – Grade 7

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 469 to 574***	***ISASP Mathematics*** ***Scale Score Range: 575 to 680***
7.G **Geometry**	**A typical student at this level can:** • identify the scale of a drawing • identify complementary, supplementary, vertical, and adjacent angles • define complementary and supplementary angles • solve problems involving lengths in 2-dimensional scale drawings • identify shapes with given conditions • identify 2-dimensional shapes resulting from planes slicing through 3-dimensional figures • solve problems involving angle measures of complementary, supplementary, vertical, and adjacent angles • apply the formulas for circumference and area of a circle in mathematical and real-world contexts, including determining the radius when given the area • find areas of triangles and special quadrilaterals and volumes of cubes and right prisms • find the area of a 2-dimensional objects composed of triangles and special quadrilaterals • find the surface area, given a net, of a 3-dimensional object with sides in the shapes of triangles and special quadrilaterals • determine the dimensions of a cube, given the surface area • find the volume of a 3-dimensional object composed of cubes and right prisms	**A typical student at this level can:** • solve area problems in 2 dimensions involving scale drawings • construct triangles given certain conditions • determine when conditions determine a unique triangle, more than 1 triangle, or no triangle • describe, compare, and contrast 2-dimensional shapes resulting from planes slicing through 3-dimensional figures • use the relationship between circumference and area to solve multistep mathematical and real-world problems • solve complex, multistep problems involving angle measures of complementary, supplementary, vertical, and adjacent angles • use definitions, properties, expressions, and equations to defend solutions in problems • find the areas, surface areas, and volumes of 2- and 3-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right rectangular and triangular prisms, and shapes composed of those prisms, and use them to solve complex, multistep mathematical or real-world problems

Iowa Mathematics PLDs – Grade 7

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 469 to 574***	***ISASP Mathematics*** ***Scale Score Range: 575 to 680***
7.SP **Statistics & Probability**	**A typical student at this level can:** • calculate and use measures of center to describe a population • determine the likelihood of an event • use random sampling to make an inference about a population • use measures of center and variability to draw comparative inferences about 2 populations • develop and use a probability model • calculate and apply simple probability in a context • compare theoretical and experimental probabilities • find probabilities of compound events with replacement • approximate the probability of a chance event by collecting data or based on a simulation • predict approximate relative frequency of a chance event given the probability	**A typical student at this level can:** • draw interpretive and comparative inferences about multiple populations • develop, use, and evaluate multiple probability models • distinguish between uniform and non-uniform probability models • compare theoretical and experimental probabilities of compound events • find probabilities of compound events without replacement

Iowa Mathematics PLDs – Grade 8

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 490 to 605***	***ISASP Mathematics*** ***Scale Score Range: 606 to 720***
8.NS **The Number System**	**A typical student at this level can:** • categorize fractions and terminating and repeating decimals as rational numbers • categorize non-repeating and non-terminating decimals, square roots of non-perfect squares, cube roots of non-perfect cubes, and π as irrational numbers • determine the decimal expansions of fractions and the fractional equivalents of terminating decimals • order irrational numbers between 2 whole numbers • write, or plot on a number line, approximations of irrational numbers	**A typical student at this level can:** • determine fractional equivalents of repeating decimals • provide general definitions of rational and irrational numbers
8.EE **Expressions & Equations**	**A typical student at this level can:** • simplify basic numerical expressions involving integer exponents • use square root and cube root symbols to represent solutions to simple equations • simplify basic expressions with square root and cube root symbols • express quantities in scientific notation • perform given operations on numbers in scientific notation • recognize the unit rate in a context represented by a linear graph as the slope of the line • apply a proportional relationship represented in a table or verbal description to determine the slope of an equation or graph • solve linear equations • identify linear equations with no solutions, 1 solution, and infinitely many solutions • interpret contexts that could be represented by linear equations or simple systems to solve problems • solve systems of linear equations by graphing or solving algebraically	**A typical student at this level can:** • simplify algebraic expressions or complex numerical expressions involving integer exponents • apply, interpret, and perform operations on numbers in scientific notation in real-world contexts • interpret graphs of proportional relationships • explain the relationship between similar triangles and the slope of a graph • interpret and analyze linear equations in 2 variables • create and solve systems of linear equations from real-world contexts • identify systems with no solutions, 1 solution, and infinitely many solutions

Iowa Mathematics PLDs – Grade 8

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 490 to 605***	***ISASP Mathematics*** ***Scale Score Range: 606 to 720***
8.F **Functions**	**A typical student at this level can:** • determine from a graph or an input-output table whether a relation is a function • evaluate linear functions • determine from a graph or equation whether a function is linear or nonlinear • create a graph of a linear function • identify key features of graphs, such as intercepts and intervals of increase • determine the slope and *y*-intercept of a linear function from a graph, an equation in slope-intercept form, 2 (*x*, *y*) values, or a verbal description • write the equation of a linear function, and use it to solve problems	**A typical student at this level can:** • define, evaluate, compare, analyze, and use functions that model nonlinear relationships between quantities in multiple representations • compare attributes of linear functions in multiple representations • interpret attributes, such as slope and intercepts, of linear functions in terms of the context • write a linear function requiring multiple decision points to model a relationship between 2 quantities, and use it to solve problems
8.G **Geometry**	**A typical student at this level can:** • identify dilations, translations, rotations, and reflections from figures or graphs • represent, and describe the effects of, transformations in the plane, given verbal or symbolic descriptions • use transformations to determine congruent or similar triangles or polygons • apply properties of triangles, interior and exterior angles, and angles formed by parallel lines and transversals to solve mathematical and real-world problems • apply the Pythagorean theorem to solve problems in right triangles • calculate volume for cylinders, cones, and spheres	**A typical student at this level can:** • analyze and justify congruence and similarity through dilations, translations, reflections, and rotations • interpret complex figures to solve multistep problems involving properties of triangles, interior and exterior angles, and angles formed by parallel lines and transversals • apply the Pythagorean theorem to find the distance between points in a 2-dimensional coordinate system and to solve problems in 3 dimensions • justify or complete a proof of the Pythagorean theorem • solve real-world problems involving volumes of cones, cylinders, and spheres

Iowa Mathematics PLDs – Grade 8

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 490 to 605***	***ISASP Mathematics*** ***Scale Score Range: 606 to 720***
8.SP **Statistics & Probability**	**A typical student at this level can:** • identify and interpret data points in scatter plots • construct and interpret a table of bivariate data • identify patterns of association, and apply trends in bivariate data • identify properties of a given linear function fitted to data, and use the graph and function to solve problems in the context of the data • estimate the slope and *y*-intercept of a line that models data in a scatter plot, and use them to write a linear function and solve problems • interpret a 2-way table summarizing data on 2 categorical variables, including describing possible associations between variables indicated by relative frequencies	**A typical student at this level can:** • interpret the slope and *y*-intercept of a linear function that models data in the context of the data • determine conditional relative frequencies in data summarized in a 2-way table

Iowa Mathematics PLDs – Grade 9

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 513 to 625***	***ISASP Mathematics*** ***Scale Score Range: 626 to 750***
HS.N **Number & Quantity**	**A typical student at this level can:** • choose units in problems and the scale and origin in graphs and data displays • interpret or apply units to solve problems • interpret the scale and origin in graphs and data displays • select, calculate, or define quantities in a given context • choose a level of accuracy appropriate to a context and limitations on measurement • simplify or perform operations on numerical or variable expressions involving whole-number or rational coefficients and whole number exponents or simple numerical or variable expressions involving rational exponents or radicals • rewrite basic numerical or variable expressions involving rational exponents and radicals • calculate approximate sums and products of 2 irrational numbers given as symbols like π or basic radicals • calculate exact sums and products of 2 rational numbers or 2 irrational numbers given as basic radicals • determine when sums and products are rational or irrational	**A typical student at this level can:** • solve problems, such as area or volume, requiring multistep unit conversions • interpret contexts to define or calculate appropriate quantities requiring multiple decision points • rewrite complex numerical or variable expressions involving rational exponents and radicals • simplify or perform operations on complex numerical or variable expressions involving rational exponents or radicals • generalize or explain the equivalence of rational exponents and radicals • rewrite, simplify, or perform operations on expressions involving rational exponents where the exponent contains a variable • calculate exact sums and products of 2 irrational numbers or 1 rational number and 1 irrational number when irrational numbers are given as radical expressions or symbols like π • apply properties of rational and irrational numbers • explain why the sum or product of two rational numbers is rational

Iowa Mathematics PLDs – Grade 9

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 513 to 625***	***ISASP Mathematics*** ***Scale Score Range: 626 to 750***
HS.A **Algebra**	**A typical student at this level can:** • identify terms and coefficients of an expression • produce an equivalent form of a linear or quadratic expression • interpret parts of a linear expression in terms of its context • rewrite parts of an expression based on its structure to reveal information about its context • factor a quadratic expression, and use factors to solve problems • add, subtract, and multiply binomials • determine equivalent forms of factorable rational expressions • identify zeros of linear and quadratic polynomials • create linear equations and inequalities, and use them to solve problems • create quadratic equations (with leading coefficient of 1) and exponential equations (with integer exponents) when given a template (such as a gravity equation), and use them to solve problems • rearrange simple formulas • given the same representation (such as a table) of two functions, distinguish between linear and exponential or linear and quadratic • solve linear equations and inequalities in 1 variable • solve quadratic equations in 1 variable by inspection (such as $x^2 = 49$) • solve quadratic equations presented in factored form and quadratic equations with integer solutions by factoring • solve a system of 2 linear equations • identify a linear equation that represents a line passing through given points • graph linear inequalities in 1 variable, linear equations and inequalities 2 variables, and the solution set to a system of linear inequalities in 2 variables • graph simple exponential (with integer exponents) and quadratic equations	**A typical student at this level can:** • produce an equivalent form of an exponential or polynomial expression • add, subtract, and multiply polynomials • interpret parts of an exponential or quadratic expression in terms of its context • interpret parts of an expression by viewing a part as a single entity • determine an appropriate form of a quadratic function to solve a problem • determine the maximum or minimum of any quadratic function with real roots • determine equivalent forms of rational expressions (including remainders in long division) • create quadratic equations (with leading coefficient greater than 1), exponential equations with rational and real exponents, and rational equations, and use them to solve problems • identify zeros of polynomials with factors provided, and use them to sketch graphs • rearrange complex formulas • provide justification for each step in solving a linear or quadratic equation • represent constraints by equations and inequalities, including systems • create and solve a system of linear equations or inequalities representing a context • solve quadratic equations with real number solutions by factoring or the quadratic formula • recognize when a quadratic equation does not have integer solutions • solve linear equations and inequalities in 1 variable with coefficients that are letters • graph exponential equations with rational and real exponents

Iowa Mathematics PLDs – Grade 9

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 513 to 625***	***ISASP Mathematics*** ***Scale Score Range: 626 to 750***
HS.F **Functions**	**A typical student at this level can:** • determine from an input-output table whether a relation is a function • identify key features of graphs, such as intercepts and intervals of increase • identify restrictions on domain and range given a context • identify the domain and range of a function given a table • identify the domain and range of a quadratic or exponential function given a graph • use appropriate function notation, and evaluate a linear, quadratic, or exponential function, or an explicitly-defined sequence, represented with function notation • recognize the rate of change of a linear function as the slope • create or identify graphs of linear functions, quadratic functions with integer roots, and simple exponential functions • identify equivalent forms of linear or quadratic functions • compare attributes of exponential functions to attributes of linear functions • identify a simple function (linear, quadratic, cubic, or exponential) that passes through given points • write a linear, simple quadratic, or simple exponential function to model a relationship between 2 quantities • extend an arithmetic or geometric sequence given as a pattern • write a sequence as an explicit formula • determine when a relationship between 2 quantities can be modeled by a linear, quadratic, or exponential function • apply a graphical representation of a linear function to solve problems • interpret parameters (such as slope and growth factor) in linear and exponential functions in terms of the context	**A typical student at this level can:** • identify the domain and range of a quadratic or exponential function given an equation • generate an explicit or recursive formula for a sequence, and translate between explicit and recursive formulas • evaluate a recursively-defined sequence represented with function notation • calculate the rate of change of a linear function in a complex context • calculate the average rate of change over an interval of a nonlinear function • apply a graphical representation of a quadratic or exponential function to solve problems • create or identify graphs of quadratic functions with real-number roots and polynomials functions when factorizations are available • identify equivalent forms of polynomial functions • rewrite a function in an equivalent form to interpret properties of the function • combine functions using arithmetic operations • identify the effect on the graph of replacing $f(x)$ with $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ • interpret properties or key features of a function to provide explanation or justification in a context • provide justification that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function

Iowa Mathematics PLDs – Grade 9

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 513 to 625***	***ISASP Mathematics*** ***Scale Score Range: 626 to 750***
HS.G **Geometry**	**A typical student at this level can:** • represent transformations in the plane given verbal or symbolic descriptions • identify transformations that do and do not preserve distance and angle • draw or identify a transformed figure, given a figure and a transformation • distinguish between a dilation and a translation, reflection, or rotation • identify arcs, angles, and segments in circles • determine the 4th vertex of a special quadrilateral, given coordinates of 3 vertices • determine the area of the base of a cylinder, given the volume • identify geometric figures based on precise definitions • given equations of lines in slope-intercept form, determine whether the lines are parallel, perpendicular, or neither • determine an equation for a line parallel or perpendicular to another line • find the midpoint of a segment and the distance between 2 points in the coordinate plane • calculate perimeters of polygons in the coordinate plane (rational side lengths) • identify a cross-section of a 3-dimensional object • use geometric shapes to describe or model real-world objects • apply rigid motions to determine if 2 figures are congruent • use transformations to determine congruent or similar triangles or polygons • perform or identify a dilation centered at the origin • apply congruence and similarity concepts and the Pythagorean theorem to solve problems • determine the volume of cylinders, cones, and spheres	**A typical student at this level can:** • provide justification to defend a geometric statement • determine the arc length and area of a sector given any central angle in degrees • explain why parallel lines have the same slope, and perpendicular lines have negative reciprocal slopes • write an equation of a parabola in vertex or standard form given focus and directrix • identify the focus and directrix of a parabola given the graph or equation in standard form • apply theorems about arcs, angles (including central, inscribed, and circumscribed), and segments related to circles • apply formulas (such as slope and distance formula) to classify a figure • find a point on a segment in the coordinate plane that divides the segment in a given ratio • calculate areas of triangles and rectangles and perimeters of polygons in the coordinate plane (irrational side lengths) • determine the volume of a pyramid • apply volume formulas for cylinders, pyramids, cones, and spheres to solve problems • apply concepts of density based on area and volume in modeling contexts • apply geometric methods in modeling or design contexts to solve problems

Iowa Mathematics PLDs – Grade 9

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 513 to 625***	***ISASP Mathematics*** ***Scale Score Range: 626 to 750***
HS.S **Statistics & Probability**	**A typical student at this level can:** • represent or interpret data in plots on a number line, and use the data to solve problems • compare and interpret center (median, mean) of 2 or more data sets • compare spread (computing interquartile range or given standard deviation) of 2 or more data sets • identify outliers • interpret differences in shape, center, and spread in the context of data, including the effects of outliers • summarize categorical data in a 2-way frequency table • identify associations and trends in data presented in a 2-way frequency table • determine joint, marginal, and conditional relative frequencies in the context of the data • identify properties of functions fitted to data, and use the functions to solve problems in the context of the data • determine a line that represents data • fit a linear function to data • interpret the meaning of slope and *y*-intercept of a linear model in the context of the data • determine unions, intersections, and complements of events, including identifying the meaning of a number in a Venn diagram • determine when two events are independent • determine the probability of an independent event	**A typical student at this level can:** • interpret marginal, joint, and conditional relative frequencies in the context of the data • use residuals to assess the fit of a linear functions • interpret the correlation coefficient of a linear fit • identify examples of relationships that are correlated and causal or correlated but not causal • apply unions, intersections, and complements, including interpreting and applying numbers in a Venn diagram to solve multistep probability problems • calculate conditional probability of A given B as P(A and B) / P(B) • calculate conditional probabilities given a two-way table • apply the Addition Rule: P(A or B) = P(A) + P(B) – P(A and B) • apply the Multiplication Rule: P(A and B) = P(A)P(B given A) = P(B)P(A given B) • use concepts of permutations and combinations in instances where $r = 1$ (such as “12 choose 1”) to solve problems

Iowa Mathematics PLDs – Grade 10

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 537 to 653***	***ISASP Mathematics*** ***Scale Score Range: 654 to 780***
HS.N **Number & Quantity**	**A typical student at this level can:** • choose units in problems and the scale and origin in graphs and data displays • interpret or apply units to solve problems • interpret the scale and origin in graphs and data displays • select, calculate, or define quantities in a given context • choose a level of accuracy appropriate to a context and limitations on measurement • simplify or perform operations on numerical or variable expressions involving whole number exponents or simple numerical or variable expressions involving rational exponents or radicals • rewrite basic numerical or variable expressions involving rational exponents and radicals • calculate approximate or exact sums and products of 2 rational numbers, 2 irrational numbers, or 1 rational number and 1 irrational number when irrational numbers are given as symbols like π or basic radicals • determine when sums and products are rational or irrational	**A typical student at this level can:** • solve problems, such as area or volume, requiring multistep unit conversions • interpret contexts to define or calculate appropriate quantities requiring multiple decision points • rewrite complex numerical or variable expressions involving rational exponents and radicals • simplify or perform operations on complex numerical or variable expressions involving rational exponents or radicals • generalize or explain the equivalence of rational exponents and radicals • rewrite, simplify, or perform operations on expressions involving rational exponents where the exponent contains a variable • calculate exact sums and products of 2 irrational numbers or 1 rational number and 1 irrational number when irrational numbers are given as radical expressions • apply properties of rational and irrational numbers • explain why the sum or product of two rational numbers is rational

Iowa Mathematics PLDs – Grade 10

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 537 to 653***	***ISASP Mathematics*** ***Scale Score Range: 654 to 780***
HS.A **Algebra**	**A typical student at this level can:** • identify terms and coefficients of an expression • produce an equivalent form of a linear or quadratic expression • interpret parts of a linear expression in terms of its context • rewrite parts of an expression based on its structure to reveal information about its context • factor a quadratic expression, and use factors to solve problems • add, subtract, and multiply binomials • determine equivalent forms of factorable rational expressions • identify zeros of linear and quadratic polynomials • create linear equations and inequalities, and use them to solve problems • create quadratic equations (with leading coefficient of 1) and exponential equations (with integer exponents) when given a template (such as a gravity equation), and use them to solve problems • rearrange simple formulas • given the same representation (such as a table) of two functions, distinguish between linear and exponential or linear and quadratic • solve linear equations and inequalities in 1 variable • solve quadratic equations in 1 variable by inspection (such as $x^2 = 49$) • solve quadratic equations presented in factored form and quadratic equations with integer solutions by factoring • solve a system of 2 linear equations • identify a linear equation that represents a line passing through given points • graph linear inequalities in 1 variable, linear equations and inequalities 2 variables, and the solution set to a system of linear inequalities in 2 variables • graph simple exponential (with integer exponents) and quadratic equations	**A typical student at this level can:** • produce an equivalent form of an exponential or polynomial expression • add, subtract, and multiply polynomials • interpret parts of an exponential or quadratic expression in terms of its context • interpret parts of an expression by viewing a part as a single entity • determine an appropriate form of a quadratic function to solve a problem • determine the maximum or minimum of any quadratic function with real roots • determine equivalent forms of rational expressions (including remainders in long division) • create quadratic equations (with leading coefficient greater than 1), exponential equations with rational and real exponents, and rational equations, and use them to solve problems • identify zeros of polynomials with factors provided, and use them to sketch graphs • rearrange complex formulas • provide justification for each step in solving a linear or quadratic equation • represent constraints by equations and inequalities, including systems • create and solve a system of linear equations or inequalities representing a context • solve quadratic equations with real number solutions by factoring or the quadratic formula • recognize when a quadratic equation does not have integer solutions • solve linear equations and inequalities in 1 variable with coefficients that are letters • graph exponential equations with rational and real exponents

Iowa Mathematics PLDs – Grade 10

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 537 to 653***	***ISASP Mathematics*** ***Scale Score Range: 654 to 780***
HS.F **Functions**	**A typical student at this level can:** • determine from an input-output table whether a relation is a function • identify key features of graphs, such as intercepts and intervals of increase • identify restrictions on domain and range given a context • identify the domain and range of a function given a table • identify the domain and range of a quadratic or exponential function given a graph • use appropriate function notation, and evaluate a linear, quadratic, or exponential function, or an explicitly-defined sequence, represented with function notation • recognize the rate of change of a linear function as the slope • create or identify graphs of linear functions, quadratic functions with integer roots, and simple exponential functions • identify equivalent forms of linear or quadratic functions • compare attributes of exponential functions to attributes of linear functions • identify a simple function (linear, quadratic, cubic, or exponential) that passes through given points • write a linear, simple quadratic, or simple exponential function to model a relationship between 2 quantities • extend an arithmetic or geometric sequence given as a pattern • write a sequence as an explicit formula • determine when a relationship between 2 quantities can be modeled by a linear, quadratic, or exponential function • apply a graphical representation of a linear function to solve problems • interpret parameters (such as slope and growth factor) in linear and exponential functions in terms of the context	**A typical student at this level can:** • identify the domain and range of a quadratic or exponential function given an equation • generate an explicit or recursive formula for a sequence, and translate between explicit and recursive formulas • evaluate a recursively-defined sequence represented with function notation • calculate the rate of change of a linear function in a complex context • calculate the average rate of change over an interval of a nonlinear function • apply a graphical representation of a quadratic or exponential function to solve problems • create or identify graphs of quadratic functions with real-number roots and polynomials functions when factorizations are available • identify equivalent forms of polynomial functions • rewrite a function in an equivalent form to interpret properties of the function • combine functions using arithmetic operations • identify the effect on the graph of replacing $f(x)$ with $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ • interpret properties or key features of a function to provide explanation or justification in a context • provide justification that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function

Iowa Mathematics PLDs – Grade 10

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 537 to 653***	***ISASP Mathematics*** ***Scale Score Range: 654 to 780***
HS.G **Geometry**	**A typical student at this level can:** • identify geometric figures based on precise definitions • represent transformations in the plane given verbal or symbolic descriptions • identify transformations that do and do not preserve distance and angle • draw or identify a transformed figure, given a figure and a transformation • distinguish between a dilation and a translation, reflection, or rotation • identify arcs, angles, and segments in circles • determine the 4th vertex of a special quadrilateral, given coordinates of 3 vertices • given equations of lines in slope-intercept form, determine whether the lines are parallel, perpendicular, or neither • find the midpoint of a segment in the coordinate plane • calculate perimeters of polygons in the coordinate plane (rational side lengths) • identify a cross-section of a 3-dimensional object • determine the area of the base of a cylinder, given the volume • use geometric shapes to describe or model real-world objects • apply rigid motions to determine if 2 figures are congruent • use congruence criteria (ASA, SAS, and SSS), AA criterion, and transformations to determine congruent or similar triangles or polygons • perform or identify a dilation centered at the origin • make or determine the accuracy of basic geometric constructions • provide justification to defend a geometric statement • apply congruence and similarity concepts and the Pythagorean theorem to solve problems • determine a given trigonometric ratio for an acute angle in a right triangle • apply theorems about arcs, angles (including central, inscribed, and circumscribed), and segments related to circles • determine the arc length and area of a sector given any central angle in degrees • determine an equation for a line parallel or perpendicular to another line • calculate areas of triangles and rectangles and perimeters of polygons in the coordinate plane • apply formulas (such as slope and distance formula) to classify a figure in the coordinate plane • find a point on a segment in the coordinate plane that divides the segment in a given ratio • find the distance between 2 points in the coordinate plane • determine the volume of cylinders, pyramids, cones, and spheres • apply concepts of density based on area and volume in modeling contexts • apply geometric methods in modeling or design contexts to solve problems	**A typical student at this level can:** • provide a complete argument to prove a geometric theorem • make or determine the accuracy of a geometric construction of an inscribed or circumscribed circle for a triangle or geometric constructions that apply more basic constructions, such as: applying the construction of the midpoint of a segment to construct the median of a triangle • apply basic trigonometric ratios and the relationship between the sine and cosine of complementary angles to solve problems • prove that all circles are similar • given 2 circles, apply similarity to determine missing radius or circumference • explain why parallel lines have the same slope, and perpendicular lines have negative reciprocal slopes • find the center and radius of a circle given by an equation in center-radius or standard form • apply the Pythagorean Theorem to derive the equation of a circle • write an equation of a parabola in vertex or standard form given focus and directrix • identify the focus and directrix of a parabola given a graph or equation in standard form • write an equation and sketch the graph of an ellipse or hyperbola given foci • identify the 3-dimensional object generated by rotating a 2-dimensional figure • apply volume formulas for cylinders, pyramids, cones, and spheres to solve problems • apply density or geometric methods in complex contexts to solve problems

Iowa Mathematics PLDs – Grade 10

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 537 to 653***	***ISASP Mathematics*** ***Scale Score Range: 654 to 780***
HS.S **Statistics & Probability**	**A typical student at this level can:** • represent or interpret data in plots on a number line, and use the data to solve problems • compare and interpret center (median, mean) of 2 or more data sets • compare spread (computing interquartile range or given standard deviation) of 2 or more data sets • identify outliers • interpret differences in shape, center, and spread in the context of data, including the effects of outliers • summarize categorical data in a 2-way frequency table • identify associations and trends in data presented in a 2-way frequency table • determine joint, marginal, and conditional relative frequencies in the context of the data • identify properties of functions fitted to data, and use the functions to solve problems in the context of the data • determine a line that represents data • fit a linear function to data • interpret the meaning of slope and *y*-intercept of a linear model in the context of the data • determine unions, intersections, and complements of events, including identifying the meaning of a number in a Venn diagram • determine when two events are independent • determine the probability of an independent event	**A typical student at this level can:** • interpret marginal, joint, and conditional relative frequencies in the context of the data • use residuals to assess the fit of a linear functions • interpret the correlation coefficient of a linear fit • identify examples of relationships that are correlated and causal or correlated but not causal • apply unions, intersections, and complements, including interpreting and applying numbers in a Venn diagram to solve multistep probability problems • calculate conditional probability of A given B as P(A and B) / P(B) • calculate conditional probabilities given a two-way table • apply the Addition Rule: P(A or B) = P(A) + P(B) – P(A and B) • apply the Multiplication Rule: P(A and B) = P(A)P(B given A) = P(B)P(A given B) • use concepts of permutations and combinations in instances where $r = 1$ (such as “12 choose 1”) to solve problems

Iowa Mathematics PLDs – Grade 11

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 559 to 674***	***ISASP Mathematics*** ***Scale Score Range: 675 to 800***
HS.N **Number & Quantity**	**A typical student at this level can:** • choose units in problems and the scale and origin in graphs and data displays • interpret or apply units to solve problems • interpret the scale and origin in graphs and data displays • select, calculate, or define quantities in a given context • choose a level of accuracy appropriate to a context and limitations on measurement • simplify or perform operations on numerical or variable expressions involving whole-number or rational exponents or radicals • rewrite numerical and variable expressions involving rational exponents and radicals • calculate approximate or exact sums and products of rational and irrational numbers • determine when sums and products are rational or irrational • determine the number of solutions to a polynomial equation • find the components of a vector, given coordinates of initial and terminal points • calculate the sum of 2 vectors, given the initial and terminal points of both • multiply a vector by a scalar • represent data in a matrix, and multiply a matrix by a scalar • add or subtract matrices, and identify when 2 matrices can be added or subtracted • calculate sums of complex numbers • know the definition of, and simplify an expression containing a power of, the imaginary number *i*	**A typical student at this level can:** • solve problems, such as area or volume, requiring multistep unit conversions • interpret contexts to define or calculate appropriate quantities requiring multiple decision points • generalize or explain the equivalence of rational exponents and radicals • rewrite, simplify, or perform operations on expressions involving rational exponents where the exponent contains a variable • apply properties of rational and irrational numbers • explain why the sum or product of two rational numbers is rational • calculate magnitude and direction of a vector • calculate the sum of 2 vectors, given the magnitude and direction of both • multiply matrices • identify when 2 matrices can be multiplied and which properties can be applied • calculate products of complex numbers • calculate and simplify expressions involving sums and products of complex numbers with a power greater than 2 • factor polynomials involving complex numbers • solve quadratic equations with real coefficients and complex solutions, giving an answer in $a + bi$ form, and relate the type of solution to the graph of the corresponding quadratic function

Iowa Mathematics PLDs – Grade 11

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 559 to 674***	***ISASP Mathematics*** ***Scale Score Range: 675 to 800***
HS.A **Algebra**	**A typical student at this level can:** • identify terms and coefficients of an expression • produce an equivalent form of a linear, quadratic, exponential, or polynomial expression • interpret parts of an expression in terms of its context • interpret parts of an expression by viewing a part as a single entity • rewrite parts of an expression based on its structure to reveal information about its context • rearrange simple formulas • factor a quadratic expression, and use factors to solve problems • determine the maximum or minimum of a quadratic function with a leading coefficient of 1 • transform expressions in exponential functions using properties of exponents • identify zeros of linear, quadratic, and cubic polynomials and polynomials with factors provided • determine equivalent forms of factorable rational expressions • add, subtract, and multiply polynomials • add, subtract, multiply, and divide rational expressions • identify a linear equation that represents a line passing through given points • distinguish between linear, exponential, and quadratic functions • create linear equations and inequalities, quadratic equations and inequalities (with leading coefficient of 1), exponential equations (with integer exponents), and simple rational equations, and use them to solve problems • represent constraints by equations and inequalities, including systems • graph linear equations and inequalities in 1 or 2 variables, exponential equations (with integer exponents), and quadratic equations • solve linear equations and inequalities and quadratic equations with real number solutions by inspection, factoring, or the quadratic formula • solve a system of 1 linear and 1 quadratic equation • solve a system of 2 linear equations • graph the solutions to a linear inequality in 2 variables and the solution set to a system of linear inequalities in 2 variables	**A typical student at this level can:** • determine an appropriate form of a quadratic function to solve a problem • determine the maximum or minimum of any quadratic function by completing the square • derive the formula for the sum of a finite geometric series, or apply the formula to solve problems • apply the Remainder Theorem to determine the remainder on division by $x - a$ prove polynomial identities • determine equivalent forms of rational expressions (including remainders in long division) • create quadratic equations and inequalities (with leading coefficient greater than 1), exponential equations with rational and real exponents, and complex rational equations, and use them to solve problems • graph exponential equations with rational and real exponents • use zeros to sketch graphs of functions defined by polynomials • rearrange complex formulas • provide justification for each step in solving a linear or quadratic equation • create and solve a system of linear equations or inequalities representing a context • solve radical and rational equations in 1 variable, and identify extraneous solutions • solve linear equations and inequalities in 1 variable with coefficients that are letters • recognize when a quadratic equation has complex solutions, and express them in the form $a \pm bi$ • solve quadratic equations in 1 variable by completing the square

Iowa Mathematics PLDs – Grade 11

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 559 to 674***	***ISASP Mathematics*** ***Scale Score Range: 675 to 800***
HS.F **Functions**	**A typical student at this level can:** • determine from an input-output table whether a relation is a function • identify the domain and range of a function given a graph or table • identify key features of graphs, such as intercepts and intervals of increase • determine when a relationship between 2 quantities can be modeled by a linear, quadratic, or exponential function • identify a simple function (linear, quadratic, cubic, or exponential) that passes through given points • distinguish between contexts modeled by linear and exponential functions • apply a graphical representation of a linear, quadratic, or exponential function to solve problems • identify restrictions on domain and range given a context • identify the domain and range of a quadratic or exponential function given an equation • use appropriate function notation, and evaluate a linear, quadratic, or exponential function, or an explicitly- or recursively-defined sequence, represented with function notation • recognize the rate of change of a linear function as the slope • create or identify graphs of linear function, quadratic functions with real number roots, exponential functions, square root functions, and polynomials functions when factorizations are available • show key features of graphs, such as intercepts, intervals of increase or decrease, relative maxima and minima, end behavior, and asymptotes • identify equivalent forms of linear, quadratic, and polynomial functions • write a linear, quadratic, exponential, or polynomial function to model a relationship between 2 quantities • extend an arithmetic or geometric sequence • write a sequence as an explicit formula • identify the effect on the graph of replacing $f(x)$ with $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ • find inverse functions • construct linear and exponential functions, including arithmetic and geometric sequences • interpret parameters in a linear or exponential function in terms of the context	**A typical student at this level can:** • generate the recursive formula for a sequence, and translate between explicit and recursive formulas • calculate the rate of change of a linear function in a complex context • calculate the average rate of change over an interval of a nonlinear function • create or identify graphs of cube root, piecewise, absolute value, logarithmic, trigonometric and rational functions • identify period, midline, and amplitude of trigonometric functions • rewrite a function in an equivalent form to interpret properties of the function • interpret properties or key features of a function to provide explanation or justification in a context • compare properties of polynomial, exponential, trigonometric, logarithmic, and absolute value functions • compose functions, and verify by composition that 1 function is the inverse of another • provide justification that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function • solve an exponential equation using logarithms with base 2, 10 or e • determine angle measures in radians, and convert angle or arc measures between degrees and radians • use special right triangles to determine the values of sine, cosine, and tangent , and apply the unit circle or Pythagorean identity • choose or construct a trigonometric function to model periodic phenomena by interpreting amplitude, frequency, and midline

Iowa Mathematics PLDs – Grade 11

	PROFICIENT	ADVANCED
	ISASP Mathematics ***Scale Score Range: 559 to 674***	***ISASP Mathematics*** ***Scale Score Range: 675 to 800***
HS.G **Geometry**	**A typical student at this level can:** • identify geometric figures based on precise definitions • represent transformations in the plane given verbal or symbolic descriptions • identify transformations that do and do not preserve distance and angle • draw or identify a transformed figure, given a figure and a transformation • distinguish between a dilation and a translation, reflection, or rotation • identify arcs, angles, and segments in circles • determine the 4th vertex of a special quadrilateral, given coordinates of 3 vertices • given equations of lines in slope-intercept form, determine whether the lines are parallel, perpendicular, or neither • find the midpoint of a segment in the coordinate plane • calculate perimeters of polygons in the coordinate plane (rational side lengths) • identify a cross-section of a 3-dimensional object • determine the area of the base of a cylinder, given the volume • use geometric shapes to describe or model real-world objects • apply rigid motions to determine if 2 figures are congruent • use congruence criteria (ASA, SAS, and SSS), AA criterion, and transformations to determine congruent or similar triangles or polygons • perform or identify a dilation centered at the origin • make or determine the accuracy of basic geometric constructions • provide justification to defend a geometric statement • apply congruence and similarity concepts and the Pythagorean theorem to solve problems • determine a given trigonometric ratio for an acute angle in a right triangle • apply theorems about arcs, angles (including central, inscribed, and circumscribed), and segments related to circles • determine the arc length and area of a sector given any central angle in degrees • determine an equation for a line parallel or perpendicular to another line • calculate areas of triangles and rectangles and perimeters of polygons in the coordinate plane • apply formulas (such as slope and distance formula) to classify a figure in the coordinate plane • find a point on a segment in the coordinate plane that divides the segment in a given ratio • find the distance between 2 points in the coordinate plane • find center and radius of a circle given by an equation in center-radius form • write an equation in vertex form of a parabola given focus and directrix • determine the volume of cylinders, pyramids, cones, and spheres • apply concepts of density based on area and volume in modeling contexts • apply geometric methods in modeling or design contexts to solve problems	**A typical student at this level can:** • provide a complete argument to prove a geometric theorem • make or determine the accuracy of a geometric construction of an inscribed or circumscribed circle for a triangle or advanced constructions that apply more basic constructions, such as: applying the construction of the midpoint of a segment to construct the median of a triangle • apply basic trigonometric ratios, the relationship between the sine and cosine of complementary angles, and the Laws of Sines and Cosines to solve problems • prove that all circles are similar • given 2 circles, apply similarity to determine missing radius or circumference • determine the arc length and area of a sector given any central angle in radians • explain why parallel lines have the same slope, and perpendicular lines have negative reciprocal slopes • find the center and radius of a circle given by an equation in standard form • apply the Pythagorean Theorem to derive the equation of a circle • write an equation of a parabola in standard form given focus and directrix • identify the focus and directrix of a parabola given a graph or equation in standard form • write an equation and sketch the graph of an ellipse or hyperbola given foci • identify the 3-dimensional object generated by rotating a 2-dimensional figure • apply volume formulas for cylinders, pyramids, cones, and spheres to solve problems • apply density or geometric methods in complex contexts to solve problems

Iowa Mathematics PLDs – Grade 11

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HS.S **Statistics & Probability**	**A typical student at this level can:** • determine when two events are independent • determine the probability of an independent event • represent or interpret data in plots on a number line, and use the data to solve problems • compare and interpret center (median, mean) of 2 or more data sets • compare spread (computing interquartile range or given standard deviation) of 2 or more data sets • identify outliers • interpret differences in shape, center, and spread in the context of data, including the effects of outliers • summarize categorical data in a 2-way frequency table • identify associations and trends in data presented in a 2-way frequency table • determine and interpret joint, marginal, and conditional relative frequencies in the context of the data • determine a line that represents data, and fit a linear function to data • identify properties of functions fitted to data, and use the functions to solve problems in the context of the data • use residuals to assess the fit of a linear functions • interpret the meaning of slope and *y*-intercept of a linear model in the context of the data • interpret the correlation coefficient of a linear fit • identify examples of relationships that are correlated and causal or correlated but not causal • determine and apply unions, intersections, and complements, including identifying, interpreting, and applying numbers in a Venn diagram to solve probability problems • use concepts of permutations and combinations in instances where $r = 1$ (such as “12 choose 1”) to solve problems	**A typical student at this level can:** • calculate standard deviation • given mean and standard deviation of a data set fit to a normal distribution, estimate population percentages • fit a quadratic, exponential, or trigonometric function to data • use residuals to assess the fit of a quadratic, exponential, or trigonometric function • calculate conditional probability of A given B as P(A and B) / P(B) • calculate conditional probabilities given a two-way table • apply the Addition Rule: P(A or B) = P(A) + P(B) – P(A and B) • apply the Multiplication Rule: P(A and B) = P(A)P(B given A) = P(B)P(A given B) • use permutations and combinations to compute probabilities of compound events • interpret the results of simulations • identify the purposes of and differences among sample surveys, experiments, and observational studies • estimate a population mean or proportion, and develop a margin of error based on sample data • identify statistical evidence needed to evaluate a claim